Multiresolution and fast decompression for optimal web-based rendering

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Abstract

Limited bandwidth is a strong constrain when efficient transmission of 3D data to Web clients and mobile applications is needed. Hence, compression methods can help in reducing the disk occupation and the bandwidth load.

However, while compression ratio is important, the use of Javascript on the Web and low power CPUS in mobile applications led to critical computational costs. Progressive decoding improves the user experience by providing a simplified version of the model that refines with time, and it's able to mask latency. Current approaches obtain this with very poor compression rates or with additional computational costs.

In this paper we present a novel multi-resolution WebGL based rendering algorithm which combines progressive loading, view-dependent resolution and mesh compression, providing high frame rates and a decoding speed of million of triangles per second in Javascript. The method is parallelizable and scalable to very large models.

Keywords: multiresolution, WebGL, 3D Web, web based 3D rendering, online 3D content deployment, mesh compression

1. Introduction

Limited bandwidth and increasing model sizes pose a challenge in the transmission of 3D data to Web clients and mobile applications. Mesh compression is a viable approach to minimize transmission time, and most research focus in this field has been on optimizing compression ratio.

Unfortunately, limited bandwidth often pairs with limited computational power, either because of Javascript environment or low CPU power mobile devices, to the point that for most algorithms decoding time becomes the bottleneck even at moderately low bandwidth. Acceptable rates can be regained reducing compression ratio (for example forfeiting connectivity compression) or using less sophisticate entropy compression al-

A different approach makes use of progressive reconstruction algorithms, which improve the user experience by providing a simplified version of the model that refines while the remaining part of the model is being downloaded. The model converges very quickly at the beginning of the download, and only the details require the full model. However this class of algorithms performs even worse in terms of decoding time (as shown in Limper [1]) or in terms of compression ratio.

Another desirable feature, especially for very large models, is view-dependent resolution: this allows to prioritize the download, decode a specific part of the model and vary resolution of the rendered geometry to maintain a constant screen resolution. This is obtained by maximizing quality at a given frame rate.

Moreover, the 3D models that are now available on the web cover a much broader range of possibilities w.r.t. the past, including point clouds, *triangle soups*, topologically complex gemetries, partially textured models. This leads to the necessity to propose a framework which could be robust enough to deal

33 with different cases.

In this paper we present a novel multi-resolution WebGL based rendering algorithm (Figure 1) which combines progresr sive loading, view-dependent resolution and a mesh compresr sion providing good rates and a decoding speed of million of triangles per second in Javascript. This method works with textured models, but it can handle non-manifold meshes, and it is also scalable to deal with very large models.

The method is based on a class of multiresolution structures [2, 3] where the "primitive" of the multiresolution is a patch made of thousands of triangles. The processing required to traverse this structure becomes a fraction of triangle based multiresolution algorithms, and allows "batch" operation on the patches: moving data from disk or network to GPU RAM, rendedleng, and decompression.

In section 3 we describe the improvement made on the multiresolution structure and how the compression algorithm was designed to optimize decoding time while maintaining a good compression ratio. In section 4 we compare it with existing web solutions for mesh compression and progressive visualization, and we analyze the performances when dealing with different classes of 3D models. It represents a solid alternative to current methods, providing a practical mean to handle 3D models on the web.

59 2. Related Work

This paper is related to several topics in the field of Computer Graphics. Among them, the main are: web-based 3D rendering, progressive and multiresolution rendering approaches,

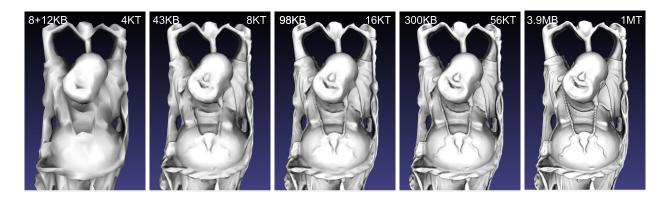


Figure 1: Progressive refinement of the Happy Buddha: on the upper left corner the size downloaded, on the upper right corner the number of triangles in the refined model. The header and index amount to 8KB

63 and fast decompression methods for 3D models.

64 While a complete overview of all these subjects goes well be-65 yound the scope of the paper, in the next subsections we provide 66 a short description of the state of the art, trying to focus on the 67 aspects which are more related to the proposed approach.

69 2.1. Web-based 3D rendering

Three-dimensional content has always been considered as 71 part of the multimedia family. Nevertheless, especially when 72 talking about web visualization, its role with respect to images 73 and videos has always been a minor one. Visualization of 3D 74 components was initially devoted to external components, such ₇₅ as Java applets or ActiveX controls [4].

After some initial efforts for standardization [5, 6], the pro-78 2.0 [8] specifications in JavaScript, brought a major change. 79 Several actions related to the use of advanced 3D graphics has 80 been proposed since then. For a general survey, please refer to 81 the work by Evans [9]. Since the use of OpenGL commands 82 needs advanced programming skills, there have been several 83 actions to provide an "interface" between them and the cre-84 ation of web pages. We could subdivide the proposed sys-85 tems between declarative approaches [10], like X3DOM [11] 124 87 SpiderGL [14] and WebGLU [15]. The main difference be-88 tween the groups is that the first ones rely on the concept of 89 scenegraph, hence a scene has to be defined in all its elements, 128 cessing the rest of the data. 90 while the second ones provide a more direct interface with the 129 91 basic commands. Other systems provide a sort of hybrid ap-92 proach [16], where a very simplified scene has to be defined. 93 Evans [9] points out in his survey that declarative approaches 94 had a major impact in the research community, while imperative 95 approaches were mainly used in the programming community. 96 More in general, given the fact that the amount of data that 97 needs to be sent to the webpage can be quite big, several efforts 98 about a better organization of generic streamable formats [17, 100 have to be streamed, these structures are not flexible enough to 101 handle them.

102 In order to face this problem, in the last three years some pro-103 gressive compression methods ad hoc for 3D streaming have 104 been developed. Gobbetti et al. [19] proposed a quad-based multi-resolution format. Behr et al. [20] transmit different quan-106 tization levels of the geometry using a set of nested GPU-friendly 107 buffers. Lavouè et al. [21] proposed an adaptation for the Web 108 (reduced decompression time at the cost of a low compression 109 ratio) of a previous progressive algorithm [22]. Other research 110 has been also conducted to handle other types of data, like point 111 clouds [23], which may present different types of issues to face

113 The rendering of textures or textured 3D models has been taken 114 into account even before the standardization actions. In these 115 cases the main issue to be faced is the amount of image data: 116 standard techniques like mip-mapping can be adapted and im-77 posal of WebGL standard [7], which is a mapping of OpenGL|ES 117 proved both on the software and hardware side [24]. The is-118 sue of handling geometric data has been usually considered a 119 minor one, due to the usual low complexity of 3D textured 120 models [25]. Nevertheless, recently complex 3D models with 121 texture coordinates are available from acquisition devices and 122 technologies. Next subsections will provide further details.

123 2.2. Progressive and Multi-resolution methods

An important feature for user experience when rendering 86 or XML3D [12], and imperative approaches, like Three.js [13], 125 over slow connections or compressed models is progressive-126 ness: the possibility to temporarily display an approximated 127 version of the model and to refine it while downloading or pro-

> The simplest (and widely used) strategy is to use a a discrete 130 set of increasing resolution models (usually known as Level Of 131 Detail, LOD). The main drawback with this approach is the 132 abrupt change in detail each time a model is replaced.

A change of paradigm was brought by progressive meshes, 134 introduced by Hoppe [26]. These meshes encode the sequence 135 of operations of a edge collapse simplification algorithm. This 136 sequence is traversed in reverse, so that each collapse becomes 137 a split, and the mesh is refined until the original resolution. An 99 18] has been proposed. Nevertheless, when complex 3D data 198 advantage of progressive techniques is the much more smooth transition resolution changes, and the possibility to combine it 140 with selective refining or view-dependent multiresolution, but sion rates: about 37 bpv with 10 bit vertex quantization.

A large number of progressive techniques were later developed, but as noted in [1], Table 1, the research focus, however, 145 was on rate-distortion performances and speed was mostly ne-146 glected. Latest algorithms still run below 200KTs in CPU.

Mobile and web application would be really too slow us-148 ing these methods. As a compromise, pop buffers [20] propose ¹⁴⁹ a method to progressively transmit geometry and connectivity, 150 while completely avoiding compression.

Another desirable feature, especially for large models, is 152 view-dependent loading and visualization. Most multiresolu-153 tion algorithms were made obsolete by the increased relative 154 performances of GPU over CPU around the first years of 2000. 155 It simply became inefficient to operate on the mesh at the level 156 of the single triangle. Several works [27, 28, 2, 29] achieved 157 much better performances by increasing the granularity of the multiresolution to a few thousand triangles.

The main problem when increasing the granularity is ensur-160 ing boundary consistency between patches at different resolu-161 tion: Yoon [27] and Sander [28] both employ a hierarchical spa-162 tial subdivision, but while the first simply disables simplifica-163 tion of most boundary edges, which results in scalability prob-164 lems, the second relies on global, spatial GPU geomorphing to ensure that progressive meshes patch simplification is consis-166 tent between adjacent blocks. The works by Cignoni [29, 2] 167 rely instead on a non hierarchical volumetric subdivision and 168 a boundary preserving patch simplification strategies that guar-169 antee coherence between different resolutions while at the same 170 time ensuring that no boundary persists for more than one level. While not progressive in a strict sense, given current rendering 172 speed, the density of triangles on screen is so high that popping 173 effects are not noticeable.

174 An additional issue when dealing with view-dependent multi-175 resolution techniques is the handling of textured models. While 176 the encoding of texture coordinates can be easily taken into 177 account when creating the patches of different resolution, the 178 boundary consistency among them needs to take into account 179 the texture images. Previous multi-resolution methods [30, 31] 180 proposed solutions for this, but they could fail when dealing with complex geometries.

Compression comes as a natural extension to this family of 183 multiresolution algorithms: each patch can be compressed independently from the others as long as the boundary still matches with neighboring patches. A wavelet based compression was developed in [32] for terrains, a 1D Haar wavelet version in [33] for generic meshes on a mobile application. A comprehensive 189 account of compression algorithms and the convergence with 190 view-dependent rendering of large datasets can be found on a 191 recent survey from Maglo et al.[34].

192 2.3. Fast Decompression of 3D models

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Given that decompression speed is a key factor in order to 194 be able to use compressed mesh, there's been some effort by the 195 community to provide solutions.

141 this high granularity was achieved at the cost of low compres- 196 Gumhold and Straßer [35] developed a connectivity only com-197 pression algorithm that was able to decompress at 800KTs in 198 1998. Pajarola and Rossignac in [36], in 2000, reported 26KTs 199 for a progressive compression algorithm, and developed a high-200 performance Huffman decoding identifying entropy compres-201 sion as a possible bottleneck.

²⁰² Finally, Isenburg and Gumhold in 2003 [37] developed a stream-203 ing approach to compression of gigantic meshes reaching an 204 impressive decompression speed of 2MTs. The method ac-205 counts also for texture coordinates. A further work on this was 206 proposed in 2005 [38].

207 3. Method

Our multiresolution algorithm builds upon the methods de-209 scribed on [2, 3], which is recapped in section 3.1 for complete-210 ness. In our solution we adopt a improved partition strategy 211 (see section 3.2), and, more importantly, a novel compression 212 scheme (section 3.3) tailored around the need for decompres-213 sion speed.

214 3.1. Batched Multiresolution

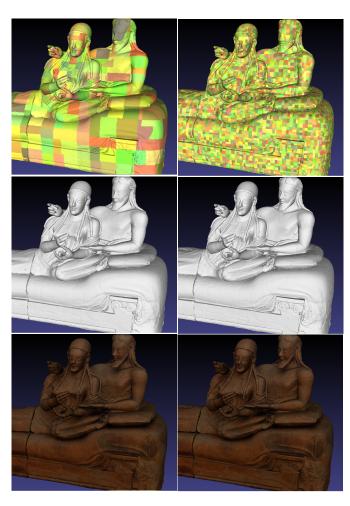


Figure 2: First column: before refinement. Second column: after refinement. From top to bottom: a visual representation of the geometric patches representing the model, the model with pure geometry, the model with color information.

The model is split into a set of small meshes at different res270 olutions that can be assembled to create a seamless mesh simply
271 traversing a tree which encodes the dependencies between each
272 patch, using the estimated screen error to select the resolution
273 needed in each part of the model. To build this collection of
274 patches we need a sequence of non-hierarchical volume par275 titions (V-partition) of the the model; non hierarchical means
276 essentially that no boundary is preserved between partitions at
277 different levels of the hierarchy.

The data structure is composed of a fixed size header describing the attributes of the models, an small index which contains the tree structure of the patches and the position of each patch in the file, and the patches themselves. We use HTTP Range requests to download header and index, ArrayBuffers to parse this structures into Javascript; the patches are then download prioritizing highest screen error. Figure 2 shows an example of a model before and after view-dependent refinement.

The rendering requires the traversal of the patch tree, which 233 is usually quite small since each patch is in the range of 16-234 32K vertices, computing the approximated screen space error 235 in pixel from the bounding sphere and the quadric error (or 236 any other error metric) during simplification. The traversal is 237 stopped whenever our triangle budget is reached, the error tar-238 get is met or the required patches are still not available.

Since the rendering can start when the first patch is downloaded and the model is refined as soon as some patch is availlable, this is effectively a progressive visualization albeit with higher granularity. On the other hand, this structure is view dependent and thus able to cope with very large models, on the order of hundreds of millions of triangles.

245 3.2. Partition

Cignoni et al [3] showed that any non-hierarchical sequence of volume partitions can be the base of a patch based multiresolution structure. Good partition strategies minimize boundaries, thus generating compact cells. In addition, they allow streaming construction and generate well balanced trees even when the distribution of the model triangles is very irregular. The Voronoi structure, while optimal for boundary minimization and balance, is not suitable for streaming, leading to long processing times. On the other hand the regular spatial subdivision used in [2] might generate unbalanced trees for very irregular models. This may impact on adaptivity.

In our solution each volume partition is defined by the leaves of a KD-tree built on the triangles of the model; to ensure the non hierarchical condition, the split ratio in the KD-tree alterates between 0.4 and 0.6 instead of the usual 0.5. This choice allows for streaming processing of the model and good adaptivity. As a bonus, the very regular shape of the patches (see figure 263 2) may be useful when adding texture support.

264 3.3. Mesh Compression

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Our multiresolution algorithm imposes a set of constrains to mesh compression:

 each patch needs to be encoded independently from the others, so the method must be efficient and fast even on small meshes

- boundary vertices, replicated on neighboring patches, need to remain consistent through compression
- non manifold models must be supported

It would be possible to exploit the redundancy of the data due to the fact that the same surface is present in patches at different levels of resolution. We choose not to do so in order to keep the compression stage independent of the simplification algorithm used and to simplify parallel decompression of the patches. Otherwise, we would have to keep track of and enforce dependencies.

280 3.3.1. Connectivity compression

We modified the algorithm presented in [39], to support non manifold meshes and surfaces with handles or holes.

We need face-face topology for compression and this is computed as follows: we create an array containing three edges for
each triangle, and sort it so that edges sharing the same vertices
will be consecutive (independently of the order of the edges).
The edges are then paired taking orientation into account, and
all non paired edges are marked as boundary. Non manifold
meshes will simply force the creation of some artificial bound-

The encoding process starts with a triangle and expands iterzest atively adding triangles. The processed region is always home-293 omorphic to a disk and if the region meets already considered triangles, we consider the common vertices as duplicated. The boundary of the already processed (encoded or decoded) region is stored as a doubly linked list of oriented edges (*active edges*), 297 The list is actually implemented as an array for performances reasons. A queue keeps track and prioritize the *active edges*.

The first triangle adds three active edges to the list; itera-100 tively an edge is extracted from the queue and, if not marked as 101 processed, the following codes are emitted (see Figure 3):

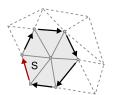
SKIP if the edge is a boundary edge, or the adjacent triangle has already been encoded; the edge is marked as processed.

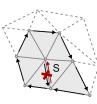
LEFT or RIGHT if the adjacent triangle shares two edges with the boundary; The two edges are marked as processed, an ew edge added to the queue and its boundary adjacencies adjusted.

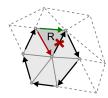
VERTEX if the adjacent triangle shares only one edge with the boundary, in this case the edges is marked as processed and two new edges added to the queue. If vertex of the new triangle opposing the edge was never encountered before its position size is estimated using parallelogram prediction and the difference encountered of the interaction and the difference its index is encoded (in literature this case is often referred as a "split"). This is a key difference with [39], where in the second case a SKIP code would be emitted, to keep the encoded region simple.

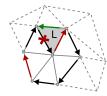
If the mesh is composed of several connected components, the process is restarted for each component.

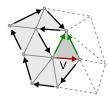
The order in which the active edges are processed is important as we would like to minimize the number of VERTEX split operations, and generate a vertex-cache-friendly triangle order. To do so, we simply prioritize the right edges in the VERTEX operation, so that the encoding proceeds in 'spirals'.











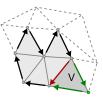


Figure 3: The four decompression codes: black arrows represent the front, the red arrow the current edge, in green the new edges added to the front.

324 If the mesh is not homeomorphic to a disk, some split operations are required. This strategy reduces the number of splits to less than 1% in our examples, incurring in an average of 0.2 bpv cost.

329 but it is extremely simple, linear in the number of triangles and 373 ployed for the vertex coordinates. 330 robust to non-manifold meshes; as we will see in the results, 374 331 speed is more important than bitrate.

332 3.3.2. Geometry and vertex attribute compression

To ensure consistency between boundary vertices of adja-334 cent patches, we adopt a global quantization grid for coordi- 378 3.3.4. Point clouds 335 nates, normals and colors. The global grid step for vertex posierrors during the simplification step in construction.

Geometry and vertex attributes are encoded as differences 338 339 to a predicted value. The distribution of these values exhibit a bias which we can exploit to minimize the number of bits necessary to encode them. Our strategy is based on the assumption that most of the bias is concentrated on the position of highest bit (the log_2 of the value) of these value while the subsequent bits are mostly random. We simply store in an array, which is later entropy coded, the number of bits necessary to encode the value; the subsequent bits are stored in an uncompressed bit- 389 3.4. Entropy coding 347 stream. In this way we need to decode a single symbol, from a 348 limited alphabet, and read a few bits from a bitstream to decode 349 a difference.

Each new vertex position, result of a VERTEX code, is es-351 timated using a simple parallelogram predictor, and the differ-352 ences with the actual position encoded as above. Color information is first converted into YCbCr color space and quantized, we encode the difference with one of the corner of the edge processed when emitting the VERTEX code. Normals vector are estimated using the decode mesh position and connectivity, and 357 differences encoded as usual.

358 3.3.3. Texture coordinates and textures

The main challenge with multiresolution textured models 360 lies in the simplification algorithm: it needs to take into ac-361 count texture seams, and minimize deformation of the texture 362 on the surface. We used the algorithm employed in Meshlab 363 [40], based on quadrics in 5 dimensions which include tex-364 ture coordinates in selection of the collapse and the vertex opti-365 mization computation. An optional step could be added which 366 would modify the texture to take into account the introduced 367 distortion as described in the work by Chen [41].

Textures coordinates in the dataset are stored per vertex, 369 replicating vertices on texture seams, while texture images are 370 stored into the dataset as JPEG binaries and loaded on demand ₃₇₁ just like the mesh patches. Finally texture coordinates are com-This algorithm is certainly not optimal in term of bitrate, 372 pressed using the same parallelogram prediction algorithm em-

> Employing more sophisticated methods would drastically 375 increase decoding time (see timings in table 2 in [42]) or re-376 quire additional linear algebra Javascript libraries, for a limited 377 decrease in bitrate.

Enabling support for point clouds requires a few changes, tion quantization is chosen automatically based on the quadric 380 merging into the same data structure, the functionality of Batched 381 Multitriangulation [3] and those of Layered Point Clouds [43]: 382 the simplification algorithm needs to be replaced with a point 383 filtering approach, where half of the points are removed at each 384 level and the compression strategy for the vertex coordinates 385 cannot rely on parallelogram prediction, in this case, after co-386 ordinate quantization we sort the points in z-order and store 387 the differences between consecutive points using the same ap-388 proach used for the meshes.

We have shown how to convert connectivity, geometry and 391 attributes into a stream of symbols and bits. It is worth com-392 pressing the symbol stream due to the biased probability distri-393 bution of the symbols.

Entropy decoding is the speed bottleneck in many mesh de-395 compression methods, often due to the main goal of minimizing 396 bit per vertex. Pajarola and Rossignac [36] developed a high-397 performance Huffman decoding algorithm in order to overcome 398 this problem. The main advantage of this method is that it re-399 duces the decoding phase to a couple of table lookups. Arith-400 metic coding, for example, outperforms Huffman in term of 401 compression rate, but exhibits lower speed. A problem with this 402 approach is the initialization time required to create the, possi-403 bly very large, decoding tables. It is then not suitable for decod-404 ing small meshes where the construction time would dominate 405 over the decoding time.

Unlike Huffman and other variable-length codes, Tunstall 407 code [44] maps a variable number of source symbols to a fixed 408 number of bits. Since in decompression the input blocks con-409 sists of a fixed number of bits and the output is a variable num-410 ber of symbols, Tunstall is slightly less efficient than Huffman, 411 especially where the bit size of the input block is small. The 412 decoding step is very similar to the high-performance Huffman 453 413 algorithm, as it consists in a lookup table and a sequence of 454 browsers on a variety of platform, from desktop machines to 414 symbols for each entry, but the table size is only determined by 455 low end cell phones. The results we report here were measured 415 the word size, and a fast method to generate it described in [45]. 456 on an iCore5 3.1Gh, using Chrome 41. Timings taken other

417 timal encoding table for a word size of N bits, we need to gen-458 tiresolution model construction, this is a preprocessing opera-418 erate 2^N symbol sequences that have a frequency as close as 459 tion. Compression time is negligible, since it can be performed $_{419}$ possible to 2^{-N} , allows to encode every possible input (it is $_{460}$ at about 1M triangles per second. he most cumbersome part is 420 complete) and no sequence is a prefix of any other sequence 461 the quadric simplification algorithm, that runs at about 60K tri-421 (it is proper).

Tunstall optimal strategy starts with the M symbols as initial 463 tion must be performed only once. $_{423}$ sequences, removes the most frequent sequence A and replaces $_{424}$ it with M sequences concatenating A with every symbol until $_{425}$ we reach 2^N sequences. The most time consuming step of the algorithm is to find the most probable sequence.

If we use a matrix where the first column contains the sorted 428 symbol in order of probability, and at each step we replace the $_{429}$ sequence with highest probability with M sequences adding a 430 new column, we can observe that this table is sorted both in 432 sequence by keeping each row in a queue and using a priority 433 queue to keep track of which queue has the highest front ele-434 ment.

Г	\rightarrow		\downarrow		\downarrow		\downarrow	
A 0.50	AA C).25	ВА	0.15	AAA	0.125	ВА	A 0.075
B 0.30	AB ().15	вв	0.09	AAB	0.075	ВА	B 0.045
C 0.10	AC (0.05	вс	0.03	AAC	0.025	BA	C 0.015
D 0.10	AD (0.05	BD	0.03	AAD	0.025	ВА	D 0.015

Figure 4: First four steps in construction of a Tunstall code with four symbols, the sequences A, B, AA, BA are replaced with a new column, beside each sequence, its probability is shown. In green the candidates for the next expansion.

To initialize the decoding table the symbol frequencies needs 485 435 436 to be transmitted in advance.

Finally, an important advantage of variable-to-fixed coding 486 438 is that the compressed stream is random accessible: decoding 487 439 can start at any block. This makes it especially suited for paral-440 lel decompression in particular GPU decompression. Unfortu-441 nately, current limitations in the capabilities of WebGL do not 442 allow for such an implementation.

443 4. Results

The testing of the proposed methods has been divided in two subsections. The first one will be related to the comparison 446 with existing Compression methods; the second one will show 447 the performances in rendering on several types of models, in 448 order to test the robustness and flexibility of the proposed com-449 pression paradigm.

A demo page, that shows the comparison and a few ex-⁴⁵¹ amples, is available at http://fastdec.duckdns.org (for reviewers 452 only)

Our implementation has been successfully tested on major Given an entropic source of M symbols, to generate an op- 457 browsers (e.g.Firefox) where comparable. Regarding the mulangles per second per core. Nevertheless, the model construc-

464 4.0.1. Comparisons with existing systems

As already stated, decompression speed is a key factor when 466 dealing with web streaming of 3D data. Hence, a tradeoff be-467 tween compression and speed has to be found to ensure optimal 468 performances.

469 For this reason, three comparisons have been performed: the 470 first one is devoted to the measurement of the decompression columns and rows (see Figure 4). This allows to select the next 471 speed of some of the reference methods. The second one fo-472 cuses on the mesh compression capabilities.

> 473 Finally, a third comparison deals with the issue of textured mod-474 els, which have been only partially taken into account by the 475 state-of-the-art method. For this reason, a test against a refer-476 ence commercial solution is shown.

4.0.2. Entropy Compression: Comparison

We tested, both in C++ and Javascript, compression rates 479 and decompression speed of:

- our implementation of Tunstall coding (T)
- Huffman coding (H), in the high-performance version of Pajarola [36] (our implementation, C++ only)
- available implementations of LZMA in C++: http://www.7-zip.org/sdk.html and Javascript: https://code.google.com/p/js-lzma/
- lz-string, a LZW based Javascript implementation http://pieroxy.net/blog/pages/lz-string/index.html

	C++			Javascript			
symbols	T	Н	LZMA	T	LZMA	LZW	
4	1058	520	1066	201	19	55	
9	369	212	170	145	10	23	
13	423	168	95	150	6	20	
17	359	136	77	163	6	19	
22	332	98	67	180	6	17	

Table 1: Decompression speed in million of output symbols per second for Poisson distribution of 32K sequences

The results are presented in Table 1, the length of 32K has 489 been chosen since it is typical in our application.

Huffman and Tunstall are very similar in term of decom-491 pression speed, the difference is mainly in the time required to

493 man, especially when increasing the number of symbols. We 538 connectivity, as they mention, is due to the dynamic memory 494 tested also other probability distributions and found little dif-495 ference in terms of speed. LZMA and LZW avoid this startup 496 cost, however their more complex and adaptive dictionary man- 540 4.0.4. Textured models 497 agement allows them to outperform Huffman and Tunstall in 541 498 term of decompression speed only for very small runs (and very 499 small dictionaries). In terms of compression ratio, Huffman and 500 LZMA performed quite close to the theoretical minimum, while 501 Tunstall was about 10% worse.

We did not implement Huffman in Javascript, as we are con-503 fident the result would be very similar. On the other hand the 504 numbers for LZMA change dramatically. Lz-string serves as a 505 comparison, as a better library, optimized for Javascript. The 506 poor LZMA performances in Javascript help explain the rela-507 tively slow performances of CTM in Limper [1].

4.0.3. Mesh Compression: Comparison

We used the Happy Budda model (in Figure 1), to compare 510 compression ratio and decompression speed with OpenCTM 511 (CTM) [46] Pop buffers (POP)[1], P3DW [21], WebGL-loader 512 (CHUN) [47]. We compare our multiresolution (OUR) and, 513 to test single resolution performances of our compression ap-514 proach, a version (FLAT) which loads only the highest reso-515 lution level of the model. In each case the model has been 516 quantized at 11 bit for coordinates and 8 bit for normals, and 517 includes colors.

	FLAT	OUR	CTM	CHUN	POP	P3DW
MB	1.9	3.9	3.5	2.8	15	4.5
bpv	28	57	51	41	220	66
full	0.4	0.9	5.3	0.06	0.5	10

Table 2: Statistics for the Happy Buddha: model size in megabytes, bit per vertex and time in seconds required to fully decompress the model.

Our decompression Javascript implementation can decode 519 about 1-3 million triangles per second with normals and colors 520 in a single thread, on a desktop machine and 0.5 MT/s on a iPhone Five. Performances are somewhat degraded when the code is run during streaming visualization.

An important comparison is with [33], which employs the same multiresolution batched strategy. For their mobile multiresolution application they reports compression rates of 45-50 bpv on large colored meshes (which should be compared to our 28bpv). The difference is probably mostly due to the different connectivity encoding which, in their case, requires 20bpv 529 against our 4 or 5bpv. It is difficult to compare the speed of 530 the two decompression approaches since they run natively in 531 C# on an iPhone4 while we run in Javascript on the same plat-532 form. Our implementation speed is still, if a bit faster than their 533 50KTS ¹, at about 60KTs. The difference is probably due their ₅₃₄ more sophisticate (and slow) arithmetic encoding.

C++ decompression speed is of course faster, reaching 9MTs, 586 536 including colors and normals, and 16MTs for just position and

492 generate the decoding tables which are much larger for Huff- 537 connectivity. The speed reported in [39] of 35KTs for just the 539 allocation in their implementation.

In the case of textured models, the compression and quan-542 tization has to be applied not only on the geometric attributes, 543 but also on the texture image.

544 Regarding the latest, using 13 bits quantization for a 4096x4096 545 pixel texture (which amount to half pixel precision) results in 546 an hardly noticeable distortion,. (see Figure 5). With parallelo-547 gram prediction, texture coordinates are encoded in about 8 bpv 548 (with a reduction of 48 to 1 respect to the standard 6 floats per 549 face). We have to include the replicated vertices coordinates, 550 that in our samples amounted to at most 15% of the total amount 551 in models with many seams. Overall adding texture coordinates 552 increases the data size of about 25%, and decompression speed 553 decreases accordingly.

We uploaded a few textured model to Sketchfab, for a comparison with a state of the art industrial solution: our multireso-556 lution structure results on average 10% smaller, although it in-557 cludes all the resolutions. Regarding the performances, please 558 refer to the examples in the demo page (http://fastdec.duckdns.org).

559 4.1. Streaming and Rendering

Loading the geometry through the Range HTTP request re-⁵⁶¹ quires an increased number of HTTP calls: one for each patch, 562 or 30-60 calls every million of triangles. This does not really 563 impact over performances: the overhead is quite small (about 564 400 bytes per call) and pipelining (the process of enqueueing 565 requests and responses between browser and server) ensures 566 full utilization of the available bandwidth. Random access is 567 really necessary only to fully exploit the view-dependent char-568 acteristics of the multiresolution structure: the code could be 569 easily modified to load the model with a single call if a higher 570 number of HTTP calls was problematic on certain web hosting 571 architectures.

In the demo page (http://fastdec.duckdns.org) it is possible 573 to compare the performances of our method w.r.t. existing solu-574 tions in the case of a slow connection. Moreover, very complex 575 geometries are also available for further testing. In the follow-576 ing, we show some example of the performances of the com-577 pression method in several types of models, in order to test its 578 flexibility.

579 4.1.1. Point clouds

Point clouds are a quite common type of models, especially 581 when large environments are taken into account. Terrestrial 582 laser canners, but also UAV may provide dense point clouds. 583 Ad-hoc solutions for encoding and rendering have been de-584 vised, but their implementation is usually very hard to be ex-585 tended to triangulated surfaces.

On the contrary, the method proposed in this paper can be seam-1887 lessly applied also in point clouds: the data that are compressed 588 and streamed are only the vertices attributes.

¹The number is extrapolated from the decoding time of a large mesh given in their paper



Figure 5: Example of textured model from left to right: texture coordinates quantized at 12 bits, at 13 bits and uncompressed. Notice the slight distorsion when precision is lower than half a pixel.

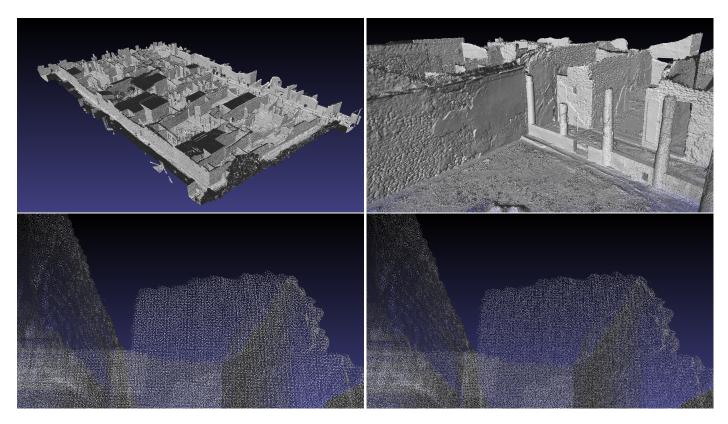


Figure 6: Pompei point cloud: original PLY file, 95M points, 2.26 Gb; uncompressed multires cloud, 1.68 Gb; compressed cloud, 326 Mb. Top left: the full compressed model, top right: a detail. Bottom left: a detail of the uncompressed point cloud. Bottom right: the same detail of the compressed point cloud



Figure 7: Bonifacio rendered in a browser: original PLY file, 84M triangles, 1.6GB; uncompressed model, 2.54 Gb; compressed model, 158 Mb. Top left: the full compressed model, top right: a detail. Bottom left: a detail of the uncompressed model . Bottom right: the same detail of the compressed model

Figure 6 shows an example of a point cloud of Insula V 591 of Pompei, obtained with terrestrial laser scanning. The top part shows the compressed model and a point of view where 593 all the details are visible. The bottom part of the Figure shows the difference between the compressed and uncompressed point 595 clouds: the quantization of the original data brings to a reduc-596 tion of points, since the quantization tends to "regularize" the 597 points grid (in this case the quantization step was 0.5 mm). This 598 effect is not noticeable in triangulated surfaces, but it may be an 599 issue for point clouds. In this case, it's possible to change the 600 quantization step in order to find the best tradeoff between com-601 pression and data quality.

603 4.1.2. Dense triangulated models

602

View-dependent progressive method have been especially 605 devised to handle dense, triangulated 3D models. For this rea-606 son, the proposed method is able to provide optimal perfor-607 mances even when hundreds million triangles have to be taken 608 into account.

Figure 7 shows the 3D model of a 3-meter tall statue which 612 was acquired with triangulation structured light scanner. The 613 compressed model, which is nearly 10 % of the original PLY 614 file, exhibits a detail that is undistinguishable from the uncom-615 pressed version.

In Figure 8 we show our system rendering the Portalada, a 618 180M triangles model at 30fps. The triangle budget has been

619 fixed at 1M triangles and the streaming requires 2-3 seconds 620 to reach full resolution on a good connection. The original 621 model is 3.6GB, while the compressed multiresolution model is 622 838MB. The Figure also shows how the view-dependent paradigm 623 is able to handle different resolutions of different parts of the 624 model when peculiar points of view are shown.

625 4.1.3. Non-optimal, topologically complicated models

Some of the solutions proposed for progressive view depen-627 dent rendering proved to be limited since their basic assump-628 tions on data processing didn't take into account that most of 629 the more complex 3D models come from acquisition devices or 630 techniques. This leads very often to the presence of geometric 631 artifacts or unbalanced density.

Figure 9 shows two examples where the method deals with 634 non-optimal geometries. On the left side, a model exhibiting 635 strong topological artifacts. On the right side, a model with 636 very unbalanced data density. In both cases, the method is able 637 to deal with the issues and provide an accurate and reliable ren-638 dering.

639 5. Conclusion

The method proposed in this paper provides good compres-641 sion ratio, progressive visualization, fast decoding and view de-642 pendent rendering. It proves effective in a wide range of band-643 width availability, computing power and rendering capabilities. 644 Moreover, it is able to handle a wide variety of 3D models types. 645 This means that also very complex geometries can be now ex-646 plored in real time with average connections speeds.

Many mesh compression algorithms for mobile and web ap-648 plication do not employ topological connectivity compression 649 often because it is believed to be excessively complex or slow 650 and limited to manifold meshes. We prove that, if implemented 651 correctly, this is not the case, and the choice of the entropy com-652 pression algorithm can play a much more important role. The 653 proposed method represents the current best tradeoff between 654 data compression and rendering performances.

655 5.1. Future improvements

The current implementation of the compression algorithm 657 has the advantage of being able to encode most of the attributes 658 of a 3D model.

Nevertheless, improvements in both compression and ren-660 dering performances can be obtained by further exploitation of In the following, two examples of complex geometries are shown. 661 the characteristics of some types of models.

> For example, in the case of Point clouds, the rendering paradigm 664 plays a key role to obtain a satisfying visualization. The current 665 rendering method could be improved by implementing and ex-666 tending existing approaches [48]. The attributes (i.e. radius) 667 that could be used for efficient rendering can be easily inserted 668 in the compression framework.

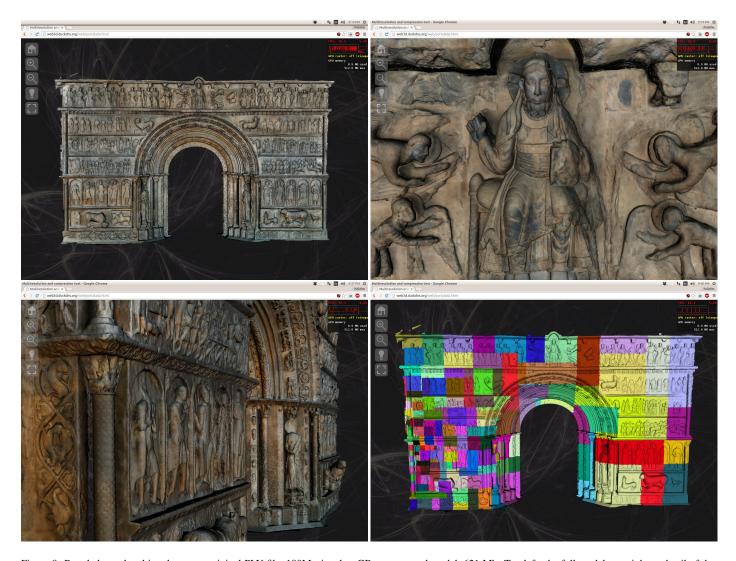


Figure 8: Portalada rendered in a browser: original PLY file, 180M triangles, GB; compressed model, 621 Mb. Top left: the full model, top right: a detail of the figure above the arch, middle right: the resolution of the model as seen from the middle left view point (without frustum culling)

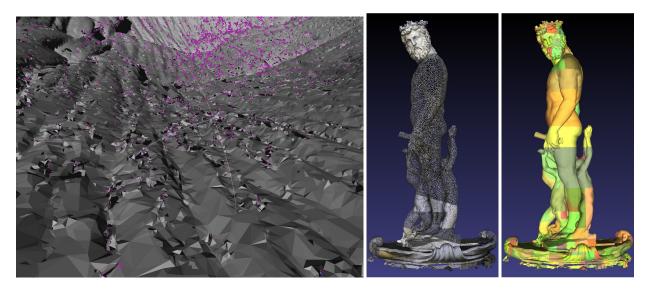


Figure 9: Left: a model with severe topological issues. Right: a model with very imbalanced vertex distribution

671 further improved, by working on ways to better compress and 672 handle textures, or moving to other texturing paradigms. An 673 example could be the projective textures (a similar approach on 734 674 point clouds was recently proposed by Arikan [49]), that could 735 675 remove the need for parametrization, and open to even more 676 complex datasets.

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